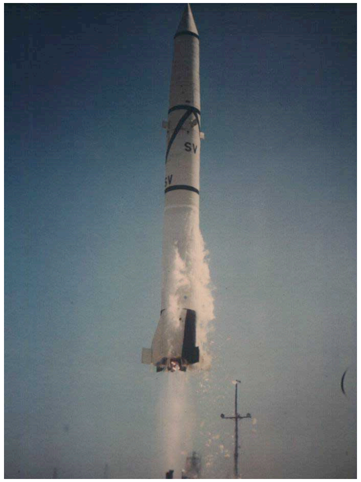
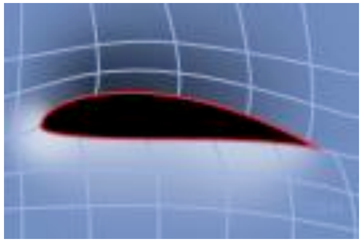
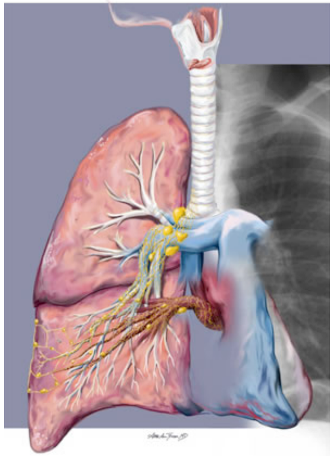
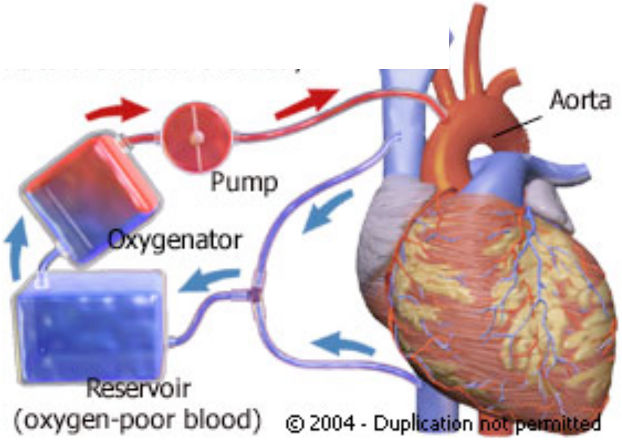
Fluids mechanics



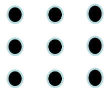


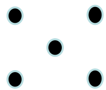


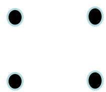


Fluid mechanics is the part of applied mechanics concerned with the behavior of liquid and gases at rest and motion

Basic characteristics of fluids

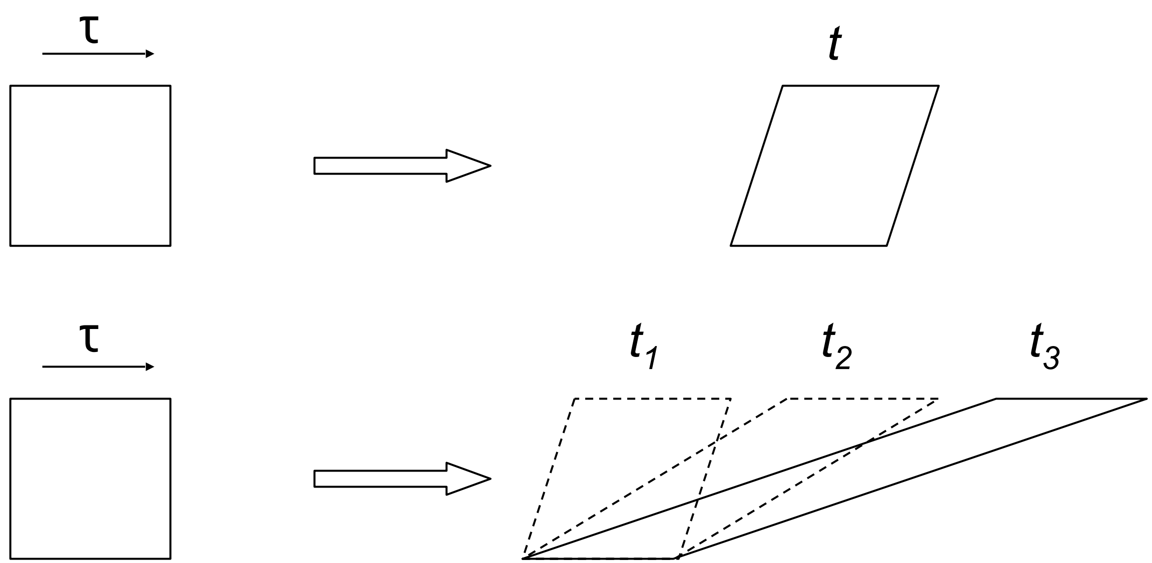
Solids  Densely spaced molecules → High intramolecular cohesive forces

Liquids  Less densely spaced molecules → Low intramolecular cohesive forces
Take the shape of the container

Gases  → No intramolecular cohesive forces
Fill any container volume they are placed

Solid

Fluid



A fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude

Analysis of Fluid Behavior

Apply:

- Newton's laws
- Conservation of energy
- 1st and 2nd law of thermodynamics

Fluid problems:
- → Fluid statics
- → Fluid dynamics

To study fluids we need to know the <u>fluid properties</u>

Fluid properties

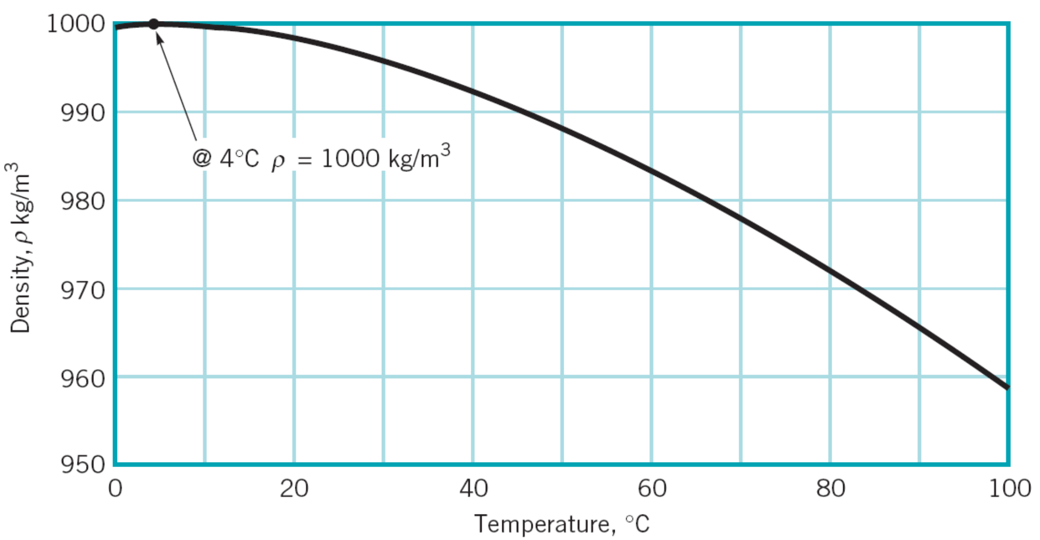
Density $\rho = \frac{mass}{volume}$ [Kg/m3]

Specific volume $v = \frac{1}{\rho}$ [m3/kg]

Specific weight $\gamma = \rho \cdot g$ [N/m3]

Specific gravity $SG = \frac{\rho}{\rho_{H2O@4^o C}}$

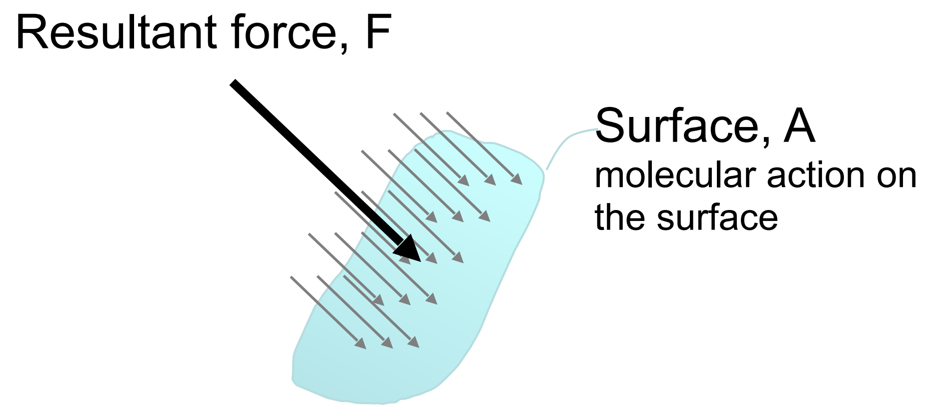
Measures of mass and weight



Density of water as a function of temperature.

Pressure

Define pressure: $P = \frac{F}{A}$



Units of pressure: N/m^2 or Pa

<u>Absolute pressure:</u> relative to absolute vacuum. Notation: (abs)
Example: P_{atm} = 101 kPa (abs)

<u>Gage pressure:</u> relative to local atmospheric pressure
Example: P_{blood} = 100 mmHg = 13.3 kPa

Ideal gas law

Liquids:

- fairly incompressible
- density changes little with pressure and temperature

Gases:

-highly compressible

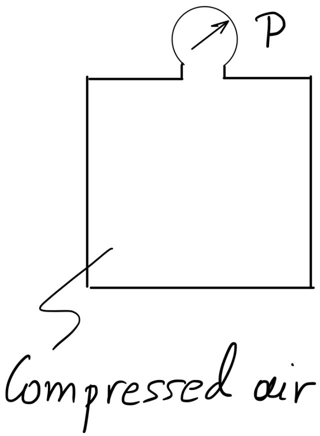
-density changes are related to pressure and temperature through the equation:

$$P = \rho R T \qquad \text{(Ideal gas law)}$$

P	**absolute** pressure
ρ	density
R	gas constant
T	**absolute** temperature

$$R = 286.9 \frac{J}{kg \cdot K}$$

Example



$m = 5\ kg$

$T = 80\ °C$

$P = 300\ kPa$

What is the volume of the tank?

Solution:

Density, $\rho = \frac{mass}{volume} = \frac{m}{V} \Longrightarrow \boldsymbol{V} = \frac{\boldsymbol{m}}{\boldsymbol{\rho}}$

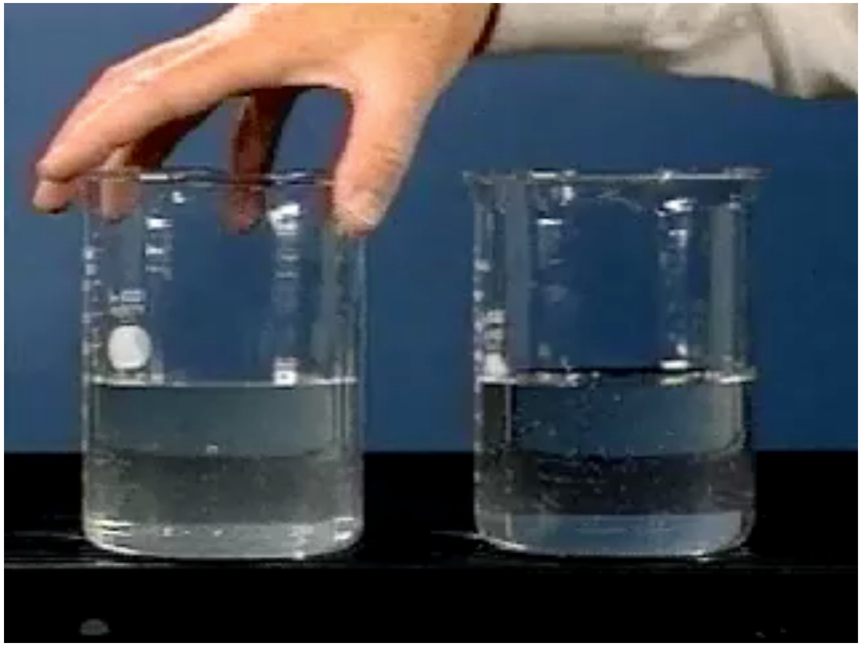
Estimate ρ using the Ideal Gas Law:

$$\rho = \frac{P}{RT} = \frac{(300 + 101)\times 10^3\ N/m^2}{(286.9\ \frac{J}{kg\ K})\times(80 + 273)K}$$

$$\Rightarrow \boldsymbol{\rho = 3.96\ kg/m^3}$$

Hence, $V = \frac{5kg}{\boldsymbol{3.96\ kg/m^3}} = 1.26\ m^3$

Fluidity



The behavior of a flowing fluid depends on various fluid properties. Viscosity, one of the important properties, is responsible for the shear force produced in a moving fluid.

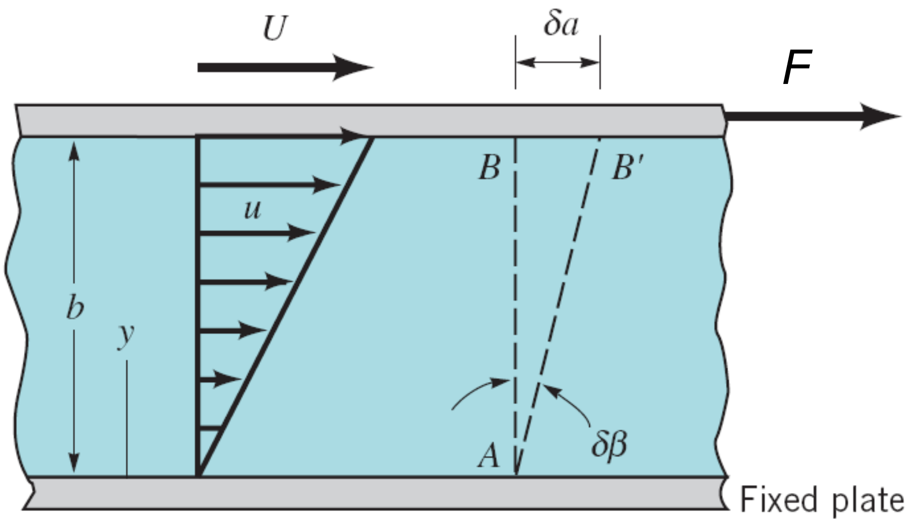
Although the two fluids shown look alike (both are clear liquids and have a specific gravity of 1), they behave very differently when set into motion. The very viscous silicone oil is approximately 10,000 times more viscous than the water

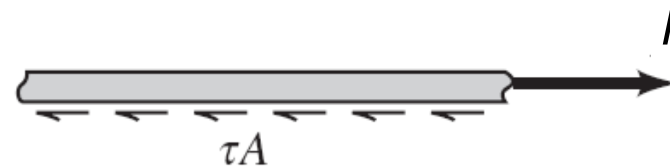
No-slip condition

As a fluid flows near a solid surface, it "sticks" to the surface, i.e., the fluid matches the velocity of the surface. This so-called "no-slip" condition is a very important one that must be satisfied in any accurate analysis of fluid flow phenomena.

Dye injected at the bottom of a channel through which water is flowing forms a stagnant layer near the bottom due to the noslip condition. As the dye filament is moved away from the bottom, the motion of the water is clearly apparent. A significant velocity gradient is created near the bottom.

Parallel plate experiment





Force balance on top plate:

$$F = \tau \cdot A$$

Experiments:

$\uparrow$F $\rightarrow$ $\uparrow$U proportionally $\rightarrow$ $\uparrow\dot{\gamma}$ proportionally

Kinematics:

$$\tan \delta\beta \approx \delta\beta = \frac{\delta\alpha}{b} = \frac{U \cdot \delta t}{b} \Rightarrow \frac{\delta\beta}{\delta t} = \frac{U}{b}$$

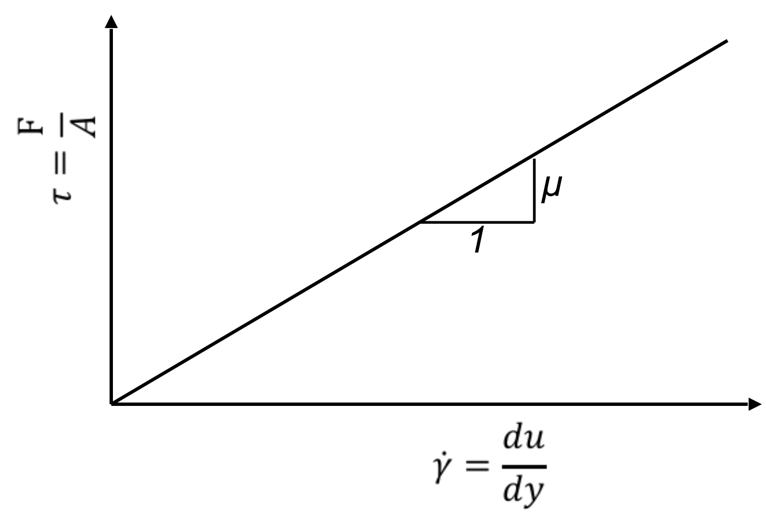
$$\tau = \mu \frac{du}{dy}$$

Velocity gradient:

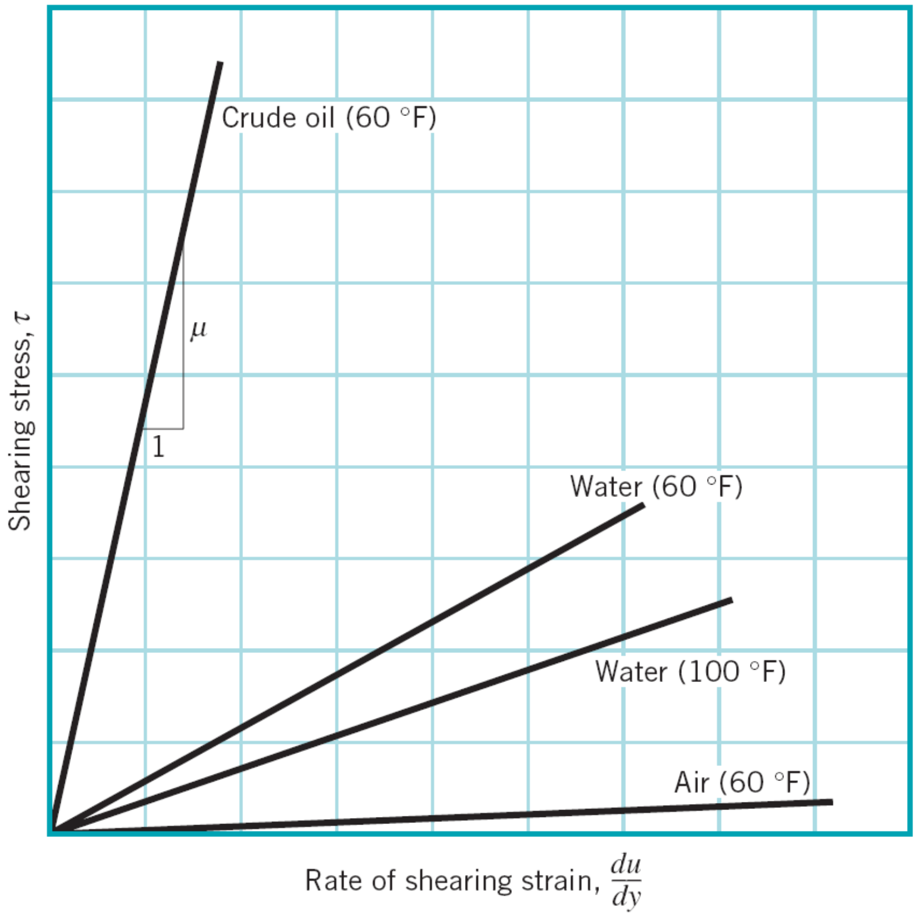
$$\frac{du}{dy} = \frac{U}{b}$$

Rate of <u>shearing strain</u>:

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\beta}{\delta t} = \frac{U}{b} = \frac{du}{dy}$$



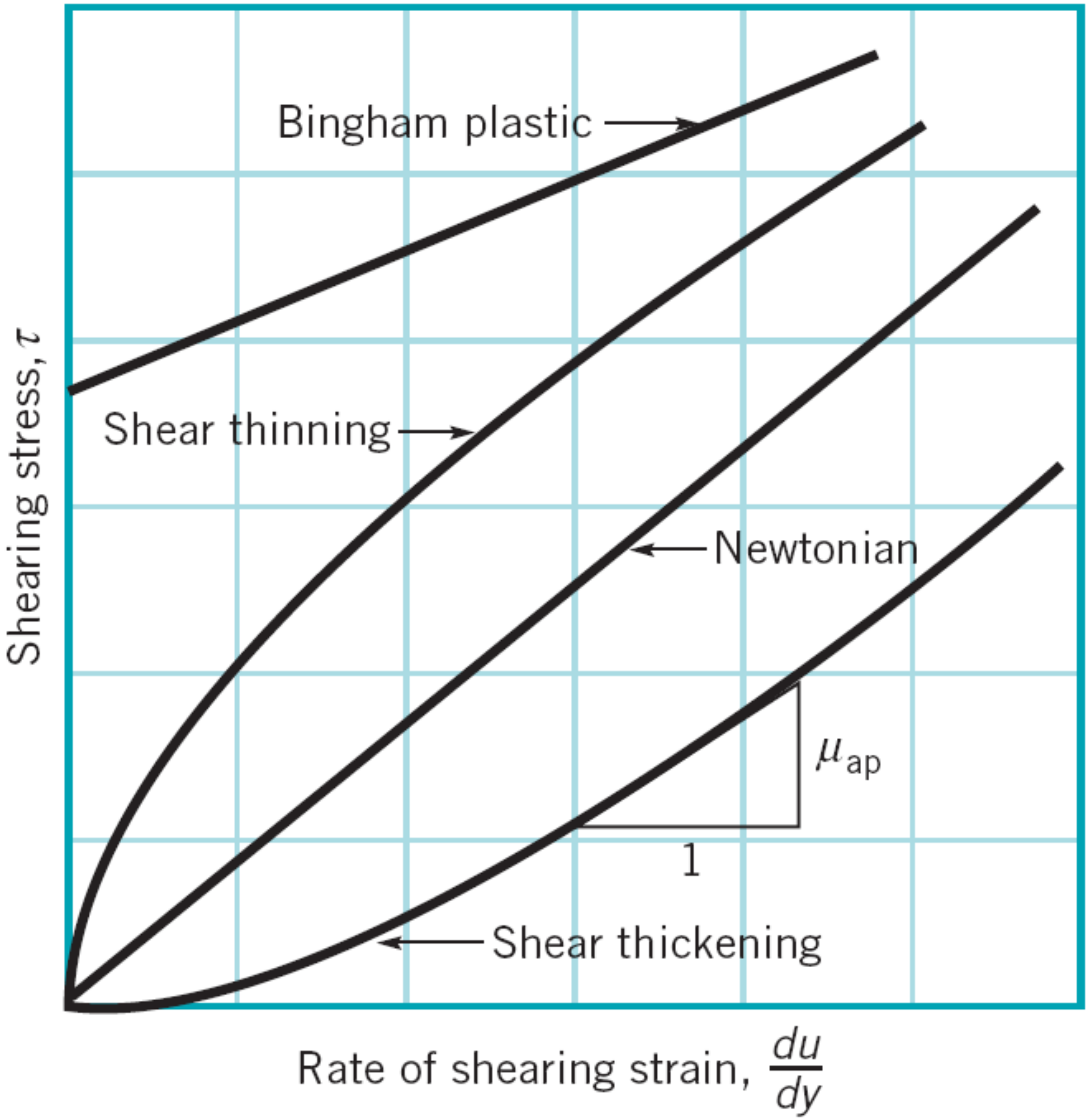
Example of common Newtonian fluids



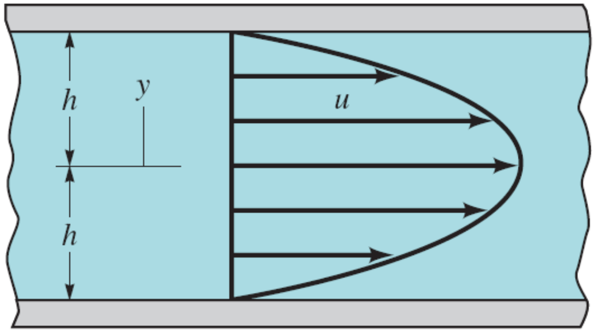
Newtonian fluids: viscosity does not depend on shearing rate

Viscosity does **not** depend on **pressure**
But it does **depend strongly on temperature**

Examples of common non-Newtonian fluids



Example: Flow between two parallel plates



$\mu = 0.05\ Ns/m^2$
V = 4 m/s
h = 15 cm

$$u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$

What is the shear stress acting on
a) The bottom wall?
b) The midplane?

$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = -\frac{3V}{h^2} y$$

a) Bottom wall: $y = -h$

$$\rightsquigarrow \frac{du}{dy} = \frac{3V}{h}$$

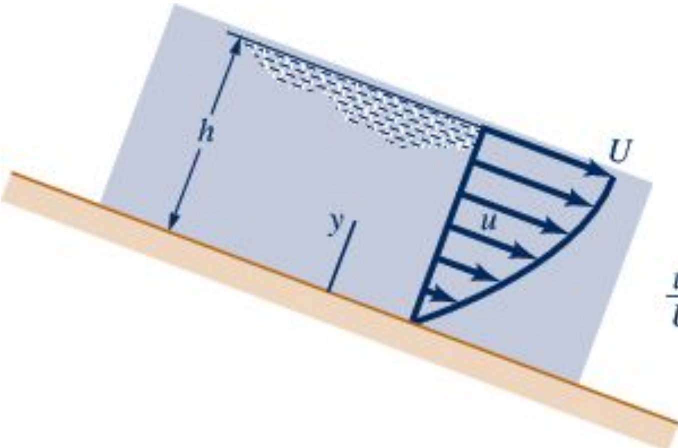
$$\tau = \mu \frac{du}{dy} = \mu \cdot \frac{3V}{h} = 0.05 \frac{N \cdot s}{m^2} \cdot \frac{3 \times 4\ m/s}{0.15\ m}$$

$$\rightsquigarrow \tau = 4\ N/m^2 \text{ or } 4\ Pa$$

b) Midplane: $y = 0 \Rightarrow \frac{du}{dy} = 0$ (symmetry)

$$\rightsquigarrow \tau_{mid} = 0$$

Example: calculation of wall shear stress



$$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$$

U = 2 m/s
h = 0.1 m

What is the magnitude and direction of the wall shear stress τ?

Solution

$$\tau_w = \mu \frac{du}{dy}\Big|_{y=0} = \mu U \left(\frac{2}{h} - \frac{2y}{h^2}\right)\Big|_{y=0}$$

$$\rightsquigarrow \quad \tau_w = \frac{2\mu U}{h} = \frac{2 \times 1.12\times 10^{-3}\ N\cdot s/m^2 \cdot 2\ m/s}{0.1\ m}$$

$$\rightsquigarrow \quad \tau_w = 4.48 \times 10^{-2}\ N/m^2$$

in the direction of flow

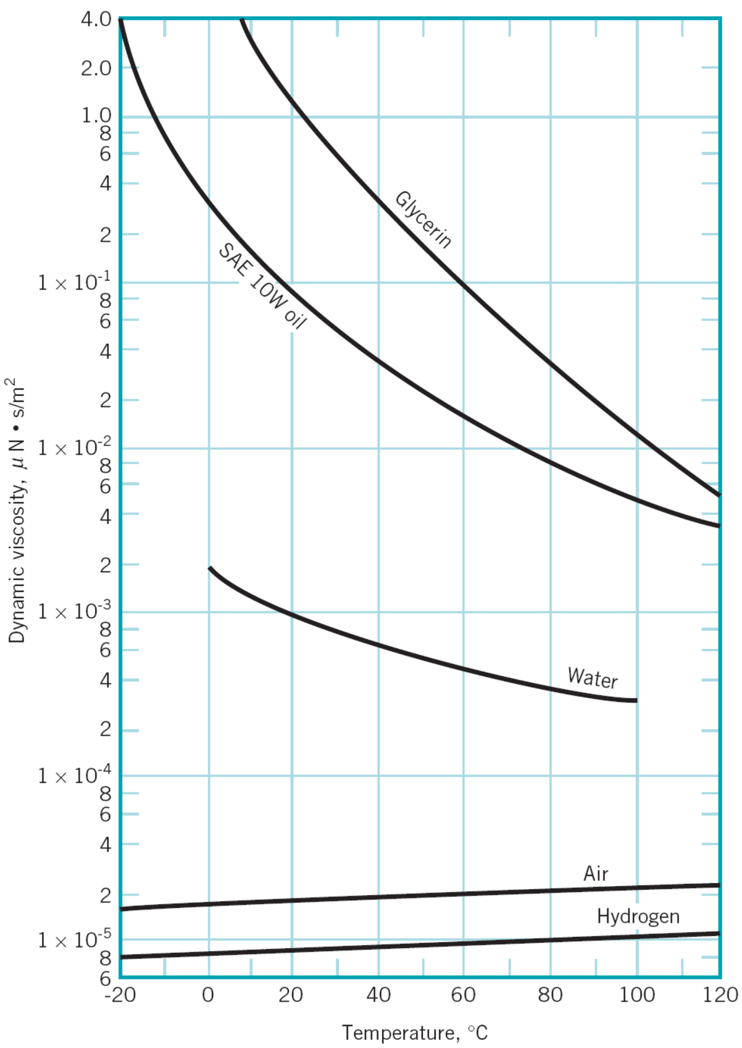
Viscosity of some common fluids as a function of temperature

<u>For gases:</u>

Sutherland equation $$\mu = \frac{CT^{3/2}}{T + S}$$

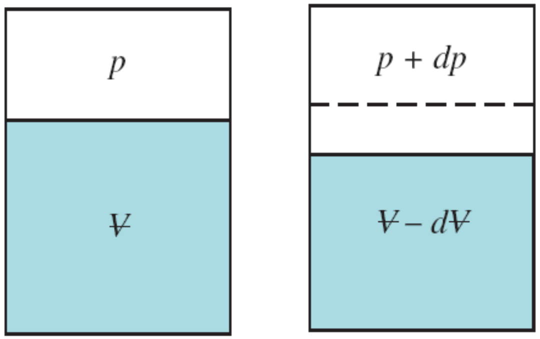
<u>For liquids:</u>

Andrade's equation $$\mu = De^{B/T}$$



Compressibility of Fluids

Liquids



Bulk modulus:

$$E_V = -\frac{dp}{dV/V} = \frac{dp}{d\rho/\rho}$$

Gases

Isothermal process:

$$\frac{p}{\rho} = \text{constant}$$

Isentropic process:

$$\frac{p}{\rho^k} = \text{constant}$$

$$k = c_P / c_V$$

$$R = c_P - c_V$$

c_p: specific heat at constant pressure

c_V: specific heat at constant volume

Speed of sound

Because of fluid compressibility, fluid disturbances (i.e., sound, etc.) travel at a certain finite speed, the **speed of sound, c.** The speed of sound is related to changes in pressure and density:

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_V}{\rho}}$$

Changes in pressure are fast, the process is **isentropic.** For **gases:**

$$\left.\begin{aligned} p = a \cdot \rho^k \Rightarrow \frac{dp}{d\rho} = a \cdot k \cdot \rho^{k-1} = k\frac{p}{\rho} \\ \frac{dp}{d\rho} = \frac{E_V}{\rho} \end{aligned}\right\} \Rightarrow E_V = kp$$

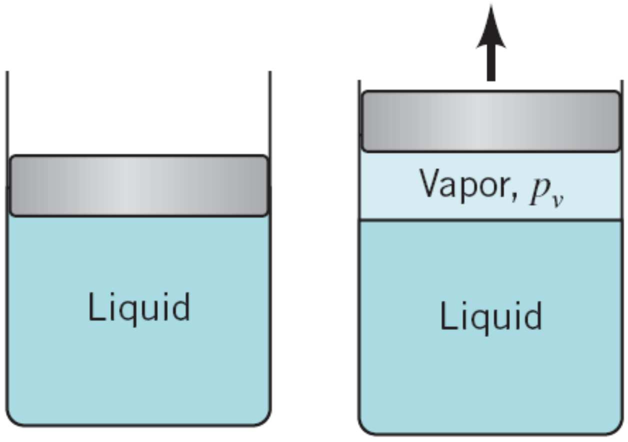
Hence,

$$c = \sqrt{\frac{kp}{\rho}} = \sqrt{kRT}$$

Air @ 15°C : $k = 1.4$ and $R = 286.9\ J/kg \cdot K$

$$\Rightarrow c_{air} = \sqrt{kRT} = \sqrt{1.4 \times 286.9 \times (273+15)}\ m/s = \underline{\underline{340\ m/s}}$$

Vapor pressure and boiling



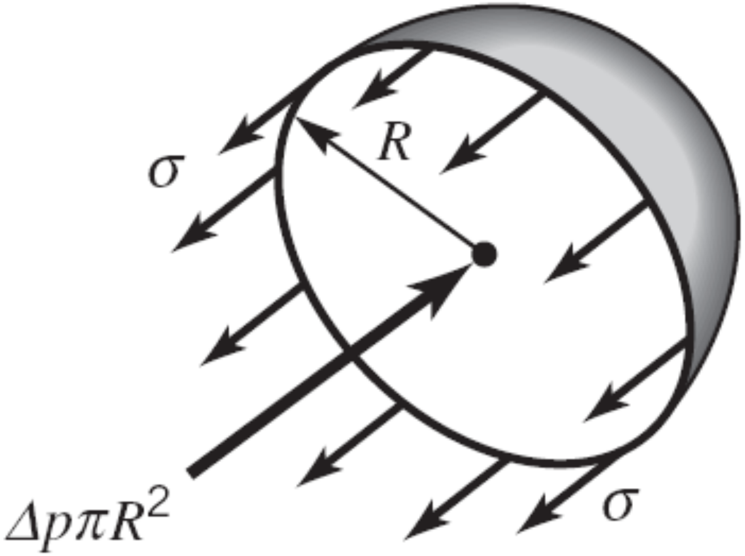
Vapor pressure depends on temperature

Boiling: formation of vapor bubbles within the fluid mass, initiated when the absolute pressure in the fluid reaches vapor pressure

To boil:

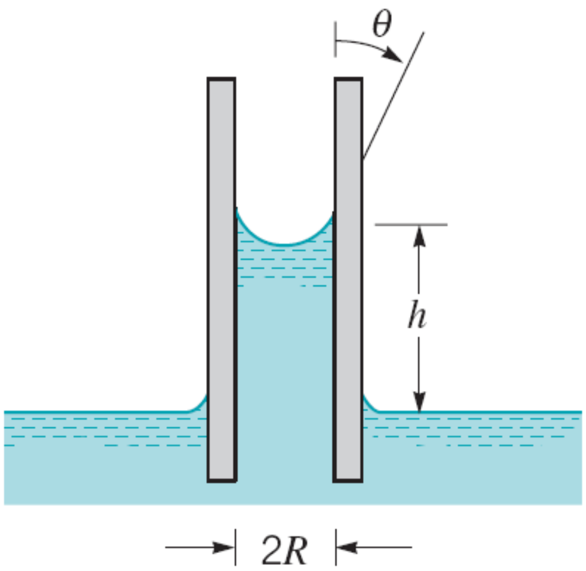
- At constant pressure, raise temperature
- At constant temperature, decrease pressure (cavitation)

Surface Tension

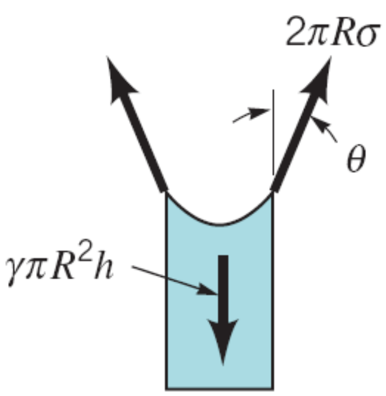


Equilibrium: $2\pi R\sigma = \Delta p \cdot \pi R^2 \quad \Longrightarrow \quad \Delta p = p_i - p_e = \frac{2\sigma}{R}$

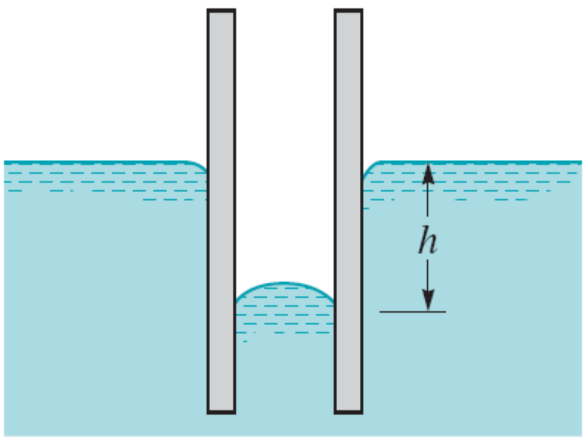
Capillary action in small tubes



Rise of column for a liquid that wets the tube.



Free-body diagram



Depression of column for a nonwetting liquid.

$$\gamma \cdot \pi R^2 \cdot h = 2\pi R\sigma \cos\theta$$

$$\Rightarrow h = \frac{2\sigma\cos\theta}{\gamma R}$$

Physical properties of liquid and gases

TABLE 1.3

Approximate Physical Properties of Some Common Liquids (SI Units)

Liquid	Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight, γ (kN/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension,[a] σ (N/m)	Vapor Pressure, p_v [N/m^2 (abs)]	Bulk Modulus,[b] E_v (N/m^2)
Carbon tetrachloride	20	1,590	15.6	9.58 E − 4	6.03 E − 7	2.69 E − 2	1.3 E + 4	1.31 E + 9
Ethyl alcohol	20	789	7.74	1.19 E − 3	1.51 E − 6	2.28 E − 2	5.9 E + 3	1.06 E + 9
Gasoline[c]	15.6	680	6.67	3.1 E − 4	4.6 E − 7	2.2 E − 2	5.5 E + 4	1.3 E + 9
Glycerin	20	1,260	12.4	1.50 E + 0	1.19 E − 3	6.33 E − 2	1.4 E − 2	4.52 E + 9
Mercury	20	13,600	133	1.57 E − 3	1.15 E − 7	4.66 E − 1	1.6 E − 1	2.85 E + 10
SAE 30 oil[c]	15.6	912	8.95	3.8 E − 1	4.2 E − 4	3.6 E − 2	—	1.5 E + 9
Seawater	15.6	1,030	10.1	1.20 E − 3	1.17 E − 6	7.34 E − 2	1.77 E + 3	2.34 E + 9
Water	15.6	999	9.80	1.12 E − 3	1.12 E − 6	7.34 E − 2	1.77 E + 3	2.15 E + 9

[a] In contact with air.
[b] Isentropic bulk modulus calculated from speed of sound.
[c] Typical values. Properties of petroleum products vary.

TABLE 1.4

Approximate Physical Properties of Some Common Gases at Standard Atmospheric Pressure (SI Units)

Gas	Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight, γ (N/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Gas Constant,[a] R (J/kg · K)	Specific Heat Ratio,[b] k
Air (standard)	15	1.23 E + 0	1.20 E + 1	1.79 E − 5	1.46 E − 5	2.869 E + 2	1.40
Carbon dioxide	20	1.83 E + 0	1.80 E + 1	1.47 E − 5	8.03 E − 6	1.889 E + 2	1.30
Helium	20	1.66 E − 1	1.63 E + 0	1.94 E − 5	1.15 E − 4	2.077 E + 3	1.66
Hydrogen	20	8.38 E − 2	8.22 E − 1	8.84 E − 6	1.05 E − 4	4.124 E + 3	1.41
Methane (natural gas)	20	6.67 E − 1	6.54 E + 0	1.10 E − 5	1.65 E − 5	5.183 E + 2	1.31
Nitrogen	20	1.16 E + 0	1.14 E + 1	1.76 E − 5	1.52 E − 5	2.968 E + 2	1.40
Oxygen	20	1.33 E + 0	1.30 E + 1	2.04 E − 5	1.53 E − 5	2.598 E + 2	1.40

[a] Values of the gas constant are independent of temperature.
[b] Values of the specific heat ratio depend only slightly on temperature.